Case-(i) In case of charged moving in the free space with no external force acting, the work done by the field acting on the charges appears as kinetic energy of the particles as

$$\frac{\partial W}{\partial t} = \sum \frac{\partial W_i}{\partial t} = \sum F_i \cdot V_i$$

ie $$\frac{\partial W}{\partial t} = \sum m_i \frac{\partial V_i}{\partial t} \cdot V_i$$

ie $$\frac{\partial W}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2} m_i V_i \cdot V_i\right)$$

$$= \sum \frac{\partial}{\partial t}\left(\frac{1}{2} m_i V_i \cdot V_i\right)$$

$$= \sum \frac{\partial}{\partial t}\left(\frac{1}{2} m_i V_i^2\right)$$

ie $$\frac{\partial W}{\partial t} = \frac{\partial}{\partial t} \sum \frac{1}{2} m_i V_i^2$$

Case -(ii)

Inside matter the work done by the field on the charge ie the kinetic energy is transfered to random motion, where it is described as heat energy or ohmic loss and is given by

$$\frac{\partial W}{\partial t} = \int_{\tau} J \cdot E \, d\tau = \int_{\tau} \frac{J^2}{\sigma} \, d\tau$$

$$\frac{\partial W}{\partial t} = \frac{J^2}{\sigma} \times S \times l$$

$$\frac{\partial W}{\partial t} = \rho \frac{I^2}{S^2} S l$$

ie $\frac{\partial W}{\partial t} = I^2 \rho \frac{l}{S}$

(B) Interpretation of $\int_\tau \frac{1}{2} \frac{\partial}{\partial t} (E \cdot D + H \cdot B) d\tau$

If we allow the volume τ to be arbitrary large surface integral in eqn (A) can be made to vanish by placing the surface S sufficiently far away so that the field cannot propagate to this distance in any finite time ie

$$\oint_S (E \times H) \cdot ds = 0$$

So under these circumastances eqn (A) reduces to

$$\frac{\partial}{\partial t} \int_{\text{all space}} \frac{1}{2} (E \cdot D + H \cdot B) d\tau + \frac{\partial W}{\partial t} = 0$$

ie $$\frac{\partial}{\partial t} \left[\int_{\text{all space}} \frac{1}{2} (E \cdot D + H \cdot B\, d\tau + W) \right] = 0$$

Thus the quantity in the square bracket is ~~reversed~~ conserved. Now consider a closed system in which the total energy is assumed to be constant. The system consists of the electromagnetic field and of all the charged particles present in the field. The term W represents the total Kinetic energy of the particles. We are therefore led to associate the remaining energy term

$$\int_{\text{all space}} \frac{1}{2}(E \cdot D + H \cdot B)\,dr$$

Withe the energy of the electromagnetic field

ie

$$U = \int_{\text{all space}} \frac{1}{2}(E \cdot D + H \cdot B)\,dr$$

the quantity U may be considered to be a kind of potentiol energy. One not need ascribe this potentiol energy to the charged particles and must consider this term as a field energy. A concept such as energy stored in the field itself rather than ~~rea~~ding with the particle in the basic concept of the theory of electromagnetism.

If we write equation

$$U = \int_{\text{all space}} u\,dr$$

$$\text{where } u = \frac{1}{2}(E \cdot D + \frac{1}{2} H \cdot B)$$

may be thought of as the energy density of the electromagnetic field

Further as

$$u = u_e + u_m$$

with $u_e = \frac{1}{2} E \cdot D$

$$= \frac{1}{2} \epsilon_r \epsilon_0 E^2 = \text{energy density of electric field.}$$

and $u_m = \frac{1}{2} H \cdot B = \frac{1}{2} \mu_r \mu_0 H^2$ = energy density of magnetic field.

(C) Interpretation of $\oint (E \times H) \cdot ds$

Instead of taking the volume integral in equations (A) over all space, let us now consider a finite volume. In this case the surface integral of $(E \times H)$ will not in general vanish and so this term must be retained. Let us construct the surface S in such a way that in the interval of time under consideration, non of the charged particles will cross this surface. Then for the conservation of energy

$$\frac{\partial U}{\partial t} + \frac{\partial W}{\partial t} = -\oint_S (E \times H) \cdot ds$$

The left hand side is the time rate of change of the energy of the field and of the particles contained within the volume γ.

Continue.